Characterizing the Exit Process of a Non-Saturated IEEE 802.11 Wireless Network

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ABSTRACT

In this paper, we consider a non-saturated IEEE 802.11 based wireless network. We use a three-way fixed point to model the node behavior with Bernoulli packet arrivals and determine closed form expressions for the distribution of the time spent between two successful transmissions in an isolated network. The results of the analysis have been verified using extensive simulations in QualNet. The methodology presented in the paper is novel and we believe that the analysis like ours can be used as an approximation to model the behavior of sub-components of a larger mesh or hybrid network.

Categories and Subject Descriptors

C.2.1 [**Computer-Communication Networks**]: Network Architecture and Design—*Wireless communication*; C.4 [**Computer Systems Organization**]: Performance of Systems—*Modeling techniques*; I.6.4 [**Computing Methodologies**]: Simulation and Modeling—*Model Validation and Analysis*

General Terms

Performance, Verification

Keywords

IEEE 802.11, bursty traffic, fixed point analysis, exit-time distribution

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MobiHoc'09, May 18–21, 2009, New Orleans, Louisiana, USA.

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1. INTRODUCTION

Indoor wireless local area networks (WLANs) based on IEEE 802.11 family of standards are by now ubiquitous and currently there is vibrant activity aimed at leveraging the low cost of IEEE 802.11 compliant products for outdoor networks. In particular, two design philosophies catering to different applications seem to be popular in the literature:

1. IEEE 802.11 based ad hoc networks for various applications like community networks [20], rescue and defense applications [16].
2. Hybrid networks where IEEE 802.11 cells are linked by a backbone mesh network comprising of potentially other technologies such as IEEE 802.16 (see [23] [2] and the references therein). Such networks are of interest to cover unserved or under-served areas.

While the interplay between coverage, user capacity, and throughput has recently been understood analytically for isolated IEEE 802.11 cells [7], for ad hoc as well as hybrid networks, as yet there is incomplete understanding of these issues. Engineering such networks is indeed an art and due to the lack of analytical tractability, detailed simulations are often the main tool. This is evidenced by the popularity of simulation tools such as NS2 [13], OPNET [14] and QualNet [17]. The simulation of such complex networks can often be significantly speeded up by resorting to analytical approximations for sub-components, whenever such good approximations are available. In this paper, we model the exit traffic from an IEEE 802.11 network. In the context of hybrid networks, this is the input traffic to the backbone mesh network, while in the context of an ad hoc network, it is the traffic leaving a cluster of nodes which can hear each other. Thus for hybrid networks, the simulation of the IEEE 802.11 WLANs at the access network level, that feeds traffic to the backbone network, can potentially be replaced by random traffic generated by our model. We note that this paper solely deals with analytical modeling of the exit traffic and the study of the hybrid network will be reported elsewhere.

In the remainder of this section, we outline the main results, compare them with prior work in this area, and describe the organization of the paper.

1.1 Main Results and Related Work

Consider a peer-to-peer system of n nodes communicating with each other using the IEEE 802.11 medium access control (MAC) protocol. We are interested in the probability distribution function of the time between two successful transmissions in the network (as seen by an external observer). If all the nodes can listen to each other, then for our purpose, we can equivalently consider this network to be the uplink of an IEEE 802.11 cell with *one* Access Point (AP), n Customer Premises Equipments (CPEs), and no downlink traffic. From this point onwards, we refer to the network as an uplink of a cell. We assume that each node has a Bernoulli arrivals of packets, which have to be communicated to the AP. The packets received by the AP are handed over to the backbone mesh network for further forwarding and we are interested in modeling the probability distribution function of the time between two successful packet receptions by the AP.

The analysis of 802.11 WLANs is well-known to be a tough problem due to the interaction of different queues via the feedback from the AP. For the saturated case, a popular and accurate analytical approach is the fixed-point analysis based on the independence assumption (also called the decoupling approximation). The independence assumption originates in the work of Bianchi [3] and it states that in steady state, the attempt processes of the various nodes are independent. Subsequently, its applicability has been extended by several authors (see for example [10], [7]) and it has been proven to be accurate for large n in [4]. We use this assumption in this paper and extend the analysis to the non-saturated case in a novel way. While the saturated case leads to relationships between the collision and attempt probabilities, in our case we also get additional relationships with the probability of the queue at a node being empty. The three way relationship can be solved numerically to obtain the desired quantities in steady state. In [11], it is suggested that for analyzing downlink TCP throughput, the saturation case results can be used with n replaced by suitable effective n. Our analysis can be used to find the effective n under different traffic conditions for the uplink set up we consider.

Based on the probabilities of collision, attempt, and empty queue, we also derive the distribution of the inter-exit times. The inter-exit time has also been referred to as service time by several researchers. The mean service time for saturation case has been derived in [6] [5] and [19]. The service time distribution has been derived for the saturation case in [26] [18] [22] and for near-saturation case in [1]. For the non-saturated case of interest to us, the mean service time is derived in [8] [15] and [12]. In [27], the authors graphically compare the observed service time distribution to several known distributions and show that exponential distribution provides a good approximation to the service time. In contrast, we analytically derive the service time distribution using a fixed-point analysis of the network. Several researchers have derived the probability generating function (PGF) for the service time in non-saturated case [24] [28] [25]. The probability density function (PDF) can be numerically computed from the PGF and a closed form for the PDF is not available. Specifically, [24] and [25] derive the average values for inter-exit times and do not provide the PDF. We derive a closed form approximation for the PDF of the inter-exit time for packets, whose parameters are obtained by fixed-point analysis. We define inter-exit time as the time observed by an external observer between two successful packet transmissions in the network. This is different from the service time of a node. Our definition of inter-exit time is similar to the ones used by the authors in [24]. Despite a plethora of literature related to IEEE 802.11 WLANs, to the best of our knowledge, a closed form expression for the probability distribution function, has not been derived. Moreover, our methodology of extending the fixed point analysis to the non-saturated case is novel.

The final justification for our simplifying assumptions and approximations is given by the close match of the analytical results with detailed QualNet simulations. We consider flows with an average arrival rate of 256 kbps, 512 kbps and 1 Mbps with the number of flows varying from 1 to 25. The IEEE 802.11b MAC is used in the simulations. Once the total load in the network reaches approximately 5.5 Mbps, the network becomes saturated. Our analytical model matches accurately with the simulation results in the non-saturated as well as the saturated regime.

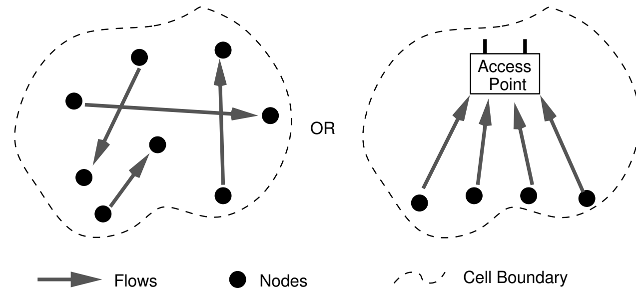


Figure 1: System Model. Both Peer-to-Peer and Access Point based uplink networks are equivalent for the sake of the analysis.

1.2 Organization of Paper

This paper is organized as follows. The system model is described in Section 2. In Section 3, we formulate the three-dimensional fixed point equation by accounting for the relationship between the System Time and Backoff Time. We also compare our analytical results with detailed QualNet simulations. Section 4 derives the service time or inter-exit time distribution. This is also compared with QualNet simulations for different network loads. Finally, we give concluding remarks in Section 5.

2. SYSTEM DESCRIPTION

We consider an IEEE 802.11 based wireless network. All nodes in the network are placed so that they are in the communication range of each other and employ a single channel for communication. Hence, one and only one transmission can occur in the network at a time. We define a *cell* as a geographical area containing the nodes in the wireless network. The cell contains n flows of uplink data traffic to the AP. The system model is shown in Figure 1. The figure shows both a peer-to-peer and an AP based system. The nodes in the cell contend using the IEEE 802.11 MAC protocol.

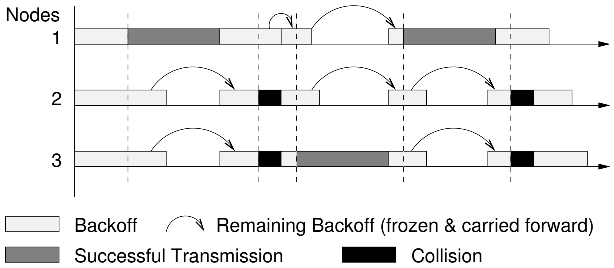


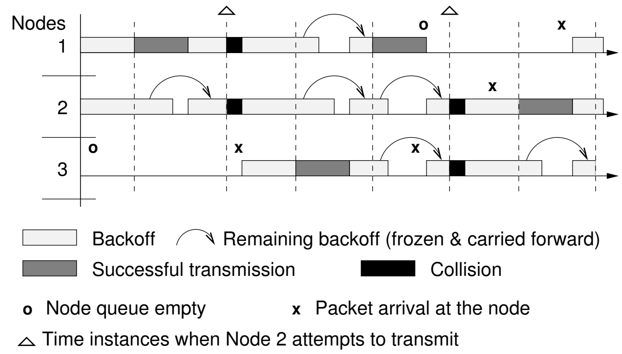
Figure 2: Aggregate Time of the System in Saturation. Backoff times are interspersed with Successful transmissions and Collisions.

The IEEE 802.11 MAC based network is a slotted system. Nodes backoff for a random number of slots using Binary Exponential Backoff (BEB) algorithm before attempting to transmit. The backoff counter is decremented by one in every time slot. When the counter reaches zero, the nodes transmit. A time slot is the minimum unit of time defined in the IEEE 802.11 MAC and for IEEE 802.11b, it has a 20 μs duration. We assume that each node in the network has Bernoulli packet arrivals. The packets leave the node when a successful transmission occurs or the maximum number of retransmission attempts are exhausted.

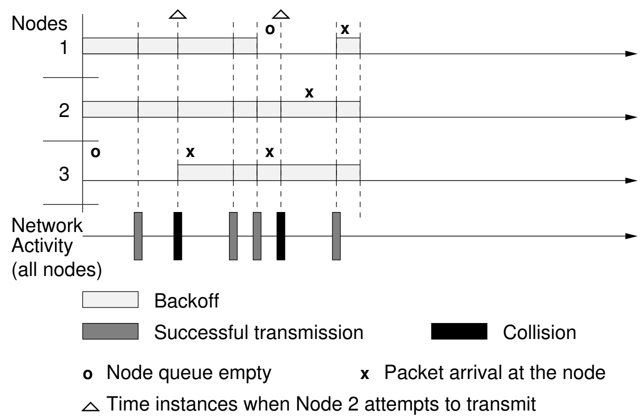
2.1 System Time and Backoff Time

The aggregate attempt process at the MAC layer for a saturated network is shown in Figure 2. It can be seen that the channel activity periods (packet transmission and collisions) do not contribute to the backoff and attempt process of a node. Also, the total time spent in backoff by all the nodes is the same. This is because, during the channel activity periods all nodes in the network, except the ones transmitting packets, freeze their backoff counters. Once the channel is free again, the nodes resume their backoff count down from the previously frozen state. The aggregate attempt process for a non-saturated network is shown in Figure 3. Unlike the saturated case, the number of contending nodes changes as a result of packet arrivals during the channel activity periods and otherwise. As seen in Figure 3(a), node 3 is not backlogged in the beginning and it participates in the channel contention only after a packet arrival. Also in the process, node 1 clears part of its backlog and at a later point in time, when it has no backlog, it no longer participates in the channel contention.

No transmission attempts are made during the channel activity periods, i.e., during packet transmissions. In the subsequent analysis, we consider System Time to represent both the time spent in channel activity (successful transmissions and collisions) and time spent in backoff as shown in Figure 3(a). The time during which the nodes backoff and count down to zero before transmissions is shown in Figure 3(b). Here, we have removed the channel activity periods and denote this time as the Backoff Time for the rest of the analysis. The network activity line in Figure 3(b) shows the result of the transmission attempts at backoff boundaries. From a node's perspective, as long as there is at least one packet in the queue, it will contend for channel access. Except for the number of backlogged nodes, the state of the nodes does not change outside the Backoff Time. Therefore, for analyzing the evolution of the states of the nodes,



(a) System Time



(b) Backoff Time, derived from the System Time shown above

Figure 3: Aggregate time of the system in a non-saturated network. The number of active nodes change depending on the queue lengths.

it is convenient to use the Backoff Time. However, for the analysis of the queue, we need the System Time. These two time scales are related through the random times spent in successful transmission and collisions. In our analysis, we account for this relationship.

3. FIXED POINT ANALYSIS OF NON-SATURATED CASE

Since all the users have the same traffic arrival rate λ and all use the same MAC parameters, they have the same performance and we can study one representative user. In this section, our goal is to determine the following three quantities:

- β = probability that a given user transmits;
- γ = probability of collision given that a packet has been transmitted;
- q_0 = probability that the queue of a given user is empty.

We derive their relationships and use them to numerically compute these quantities. In subsequent sections, these are used to derive the inter-exit time distribution. We note that while β and γ are determined by the dynamics during the Backoff Time, the queue at each node evolves in System

Time, and we need to account for these two times. The additional parameters required for this are summarized in Table 1.

Table 1: Notation

Description	Symbol
Number of contending nodes	n
Minimum contention window	CW_{min}
Maximum contention window	CW_{max}
Maximum number of retries for a packet	k
Effective arrival rate in Backoff Time	λ_{BO}
Slots required by successful transmission	T_s
Slots required by collision transmission	T_c

The section is organized as follows. In Section 3.1, we derive expressions for γ and β and in Section 3.2 we analyze q_0. The overheads are computed for IEEE 802.11 in Section 3.3 and in Section 3.4 we compare our results with QualNet simulations.

3.1 Calculation of Attempt Rate and Collision Probability

We follow the standard method of [10] with a variation to account for q_0, which is greater than zero in the non-saturated case. Let R denote the number of attempts needed to transmit a packet and k be the maximum retries for a packet. Then the average number of attempts required to transmit a packet can be calculated as $\mathbf{E}[R] = 1 + \gamma + \gamma^2 + \cdots + \gamma^k$ and the average time spent in backoff before an attempt is $\mathbf{E}[X] = b_0 + b_1\gamma + b_2\gamma^2 + \cdots + b_k\gamma^k$. Here, $b_i = \frac{2^i CW_{min}}{2}$, for backoff stage i. The value for b_i is limited by the maximum number of retries (k) and the maximum contention window CW_{max}. After each transmission, the node repeats the procedure to transmit the packet. Hence, each attempt can be treated as an independent and identical process. The number of attempts to transmit, R, can be viewed as a 'reward' associated with the renewal cycle of length X [10]. Hence, the renewal reward theorem yields

$$\beta = \frac{1 + \gamma + \gamma^2 + \cdots + \gamma^k}{b_0 + b_1\gamma + b_2\gamma^2 + \cdots + b_k\gamma^k}. \quad (1)$$

Now, based on the decoupling assumption, the other backlogged nodes in the network attempt with a rate β independently of the given node. The probability that a node is backlogged is $(1 - q_0)$. The probability that an attempted transmission fails is

$$\gamma = 1 - P(\text{None of the other } n-1 \text{ attempt})$$
$$= 1 - \sum_{l=0}^{n-1} \left[\binom{n-1}{l} q_0^{n-1-l} (1-q_0)^l (1-\beta)^l \right]. \quad (2)$$

3.2 Calculation of q_0

The analysis of β and γ is based on the time spent only in the Backoff Time by nodes. The number of backlogged nodes is relevant for the backoff process only during the Backoff Time as shown in Figure 3(b). However, arrivals to the nodes can happen during the Backoff Time as well as the successful transmissions and collisions. So, the backlogged status of nodes changes due to arrivals that occur in the System Time as shown in Figure 3(a). Note that λ denotes the rate of arrival of packets in System Time and β and γ are calculated in Backoff Time. Hence, for the analysis of the queue lengths, we assume that all arrivals happen only during the Backoff Time. For this, we need to account for the arrivals occurring during channel activity periods and assume them to happen during the Backoff Time. We denote this effective arrival rate by λ_{BO}. By doing this, now we can analyze all the activity in the network in the Backoff Time. We next determine the effective arrival rate, which is then used to determine q_0.

As shown in Figure 3(a), let the constant time spent in a successful transmission and a collision be T_s and T_c respectively. If we consider a finite time window, then the relationship between the System Time and the Backoff Time depends on the random number of packet transmissions, failures and successes. To simplify the analysis, we consider the spirit of the "mean-field" approximation [4] - the aggregate behavior of the network as seen by a single user is replaced by the mean behavior. Thus, we use the conditional expectation of the System Time given the Backoff Time. The final justification for this step, as for our other approximations, is the close match we get with QualNet simulations. The conditional mean of System Time between two attempts = (T_c · Avg. No. of Collisions in y slots) + (T_s · Avg. No. of Successes in y slots) + y slots, where y is the number of slots between two attempts in Backoff Time for the tagged node (see events marked with a triangle for node 2 in Figure 3). The number of attempts by all the other nodes is binomially distributed in y slots. The probability of attempt in a slot by any backlogged node is given by $1 - (1-\beta)^{n^*}$, where $n^* = (n-1)\cdot(1-q_0)$ is the number of backlogged nodes in the network. (We note that we have once again replaced the number of contending users by the mean.) Hence, the mean number of attempts in y slots can be written as

$$L \;=\; y \cdot \left[1 - (1-\beta)^{n^*}\right]. \quad (3)$$

As we are aware that the given an attempt, the probability of collision is γ. We can say there are, on an average, $\gamma \cdot L$ collisions and $(1-\gamma)\cdot L$ successes in the System Time between two attempts by a tagged node. So the conditional mean of the System Time is given by

$$\big(T_c(\gamma \cdot L)\big) + \big(T_s(1-\gamma)L\big) + y \text{ slots}$$
$$= L \cdot \left[T_c\gamma + T_s(1-\gamma)\right] + y \text{ slots}$$
$$= \left[\left(1 - (1-\beta)^{n^*}\right)\left(T_c\gamma + T_s(1-\gamma)\right) + 1\right] y.$$

Thus the conditional mean of the System Time given that the Backoff Time is y, is a multiple of y. We use this scaling factor to define the effective arrival rate:

$$\lambda_{BO} = \frac{\lambda}{\left(1 - (1-\beta)^{n^*}\right)\left(T_c\gamma + T_s(1-\gamma)\right) + 1}. \quad (4)$$

Having determined the effective arrival rate, we now determine q_0. The queues at each node evolves as discrete time birth-death process. We have Bernoulli arrivals with rate λ_{BO} at each node. The packets leave the node on successful transmission. The probability of a birth is $\lambda_{BO}(1-[\beta(1-\gamma)])$ and the probability of death (for nonzero queue length) is $\beta(1-\gamma)(1-\lambda_{BO})$. From [9], we obtain

$$q_0 = 1 - \frac{\lambda_{BO}(1 - [\beta(1-\gamma)])}{\beta(1-\gamma)(1-\lambda_{BO})}. \quad (5)$$

Remark: The equations (1), (2), and (5) can be viewed a 3-dimensional fixed point equation in terms of β, γ, q_0. While Brouwer's fixed point theorem [21] gives the existence of a solution, in general the function involved is not a contraction. Hence to solve this equation we first fix a q_0, iterate between (1) and (2) several times to determine $\beta(q_0)$, $\gamma(q_0)$. Then we update q_0 using (5). This process is continued till numerical convergence is observed.

3.3 Determining Transmission Times: T_s, T_c

The parameters considered for computing the fixed overheads of T_s and T_c for successful transmission and collision are given in Table 2.

Table 2: Time Consumed in IEEE 802.11 Packet Transmission

Description	Time
Packet Size for data flows	1500 bytes
PHY Data Rate of IEEE 802.11	11 Mbps
Slot time	20 μs
DIFS (T_{DIFS}) (1.5 slots)	50 μs
SIFS (T_{SIFS}) (0.5 slots)	10 μs
PHY Layer overhead (T_{PHY})	192 μs
Time to transmit RTS - 20 byte (T_{RTS})	207 μs
Time to transmit CTS - 14 byte (T_{CTS})	203 μs
Time to transmit ACK - 14 byte (T_{ACK})	203 μs
Time to transmit DATA - (T_{DATA})	1112 μs
Time for CTS/ACK Timeout - (T_{TO})	408 μs

Since the calculations for β, γ, q_0 and λ_{BO} are in terms of time slots, we will convert the transmission and collision times to time slots. If a time slot is represented by τ, the respective number of slots for the transmission and collisions are as follows.

For Basic access mechanism (DATA-ACK):

$$T_s = (1/\tau)\big(T_{DIFS} + T_{PHY} + T_{DATA} + T_{SIFS} + T_{ACK}\big)$$

$$T_c = (1/\tau)\big(T_{DIFS} + T_{PHY} + T_{DATA} + T_{TO}\big)$$

For Distributed Coordination Function (DCF) mechanism (RTS-CTS-DATA-ACK):

$$T_s = (1/\tau)\big(T_{DIFS} + T_{PHY} + T_{RTS} + T_{SIFS} + T_{CTS} + T_{SIFS} + T_{DATA} + T_{SIFS} + T_{ACK}\big)$$

$$T_c = (1/\tau)\big(T_{DIFS} + T_{PHY} + T_{RTS} + T_{TO}\big)$$

With the IEEE 802.11b parameters, these times are computed to be 78 and 88 slots for T_s and T_c respectively in Basic mode of access. In DCF mode of access, these times are 101 and 44 slots for T_s and T_c respectively.

3.4 Comparison with Simulations

In this section, we compare our analysis with simulations. The simulations have been performed in QualNet network simulator [17]. We consider a single-cell IEEE 802.11b based network for simulations. The parameters used are given in Table 3. We repeat the expirements for five different seed values and plot the average values obtained. The Figures 4-7 show the average values across five runs with 95% confidence interval. We use the Variable Bit Rate (VBR) traffic generator of QualNet which generates traffic with Exponential inter-arrival times with the desired mean interval time.

Table 3: Parameters used in Simulations and for the Analytical Model

Parameters	Values
Cell size	250 x 250 meters
Number of flows	1 to 25 in steps of 1
Packet size	1500 bytes
MAC	IEEE 802.11
PHY Data Rate	11 Mbps
RTS Threshold	0 bytes (for DCF mode)
Long Retry Limit (k)	7
Duration of flows	300 s (Start:0s & End: 300s)
Rate of each flow	256 kbps, 512 kbps & 1 Mbps

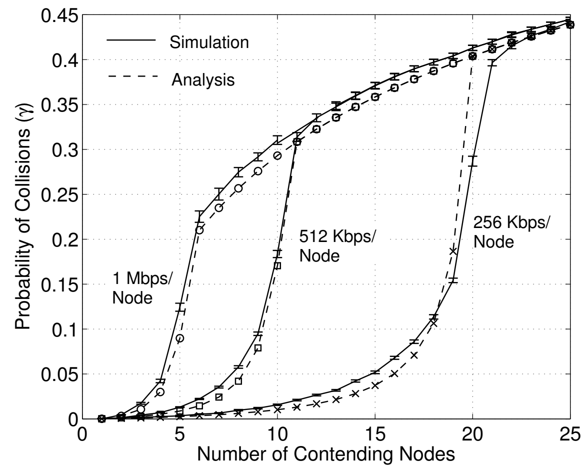


Figure 4: Comparison of Collision Probability for different arrival rates

Static routing has been used in the simulations to avoid periodic routing updates initiated by the routing protocols. Hence, the only packets transmitted by the nodes are the ones generated by the VBR application. RTS threshold is set to the zero, so all packets being transmitted require a RTS-CTS exchange under the DCF mode of operation.

The slow convergence time of the queues can lead to bias in the statistics if the simulation duration is not large. To reduce the convergence time and expedite the simulations, we initialize the nodes with non-zero queue occupancy at the start of the simulation. We choose the distribution of initial queue length to be geometric with parameter $(1 - q_0)$ taken from the analytical model. This helps the queues reach steady state faster and with relatively shorter duration of simulation we get accurate results.

Statistics about the packet transmissions, collisions, buffer occupancy at nodes and packet arrivals are collected during the simulations to obtain the collision probability and queue lengths. The queue length at each node is periodically logged for the entire duration of the simulation. We choose the interval for logging at 50 msec. At the end of the simulation, the number of instances of queue length being empty is divided by the total number of log entries to get the probability of queue being empty. All the packet transmission attempts are logged and the ratio of number of unsuccessful attempts to the total number of attempts by a node gives the collision probability.

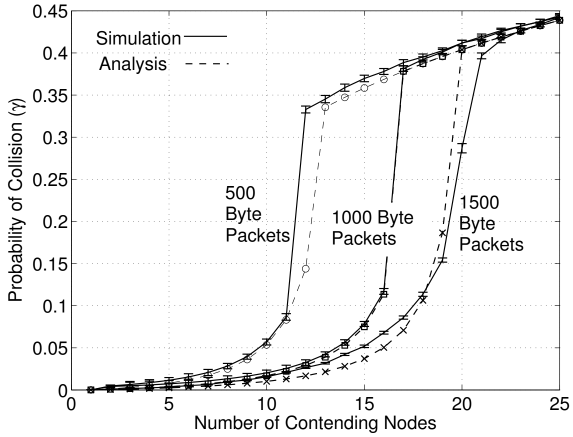


Figure 5: Comparison of collision probability for varying packet sizes at 256 kbps traffic per node

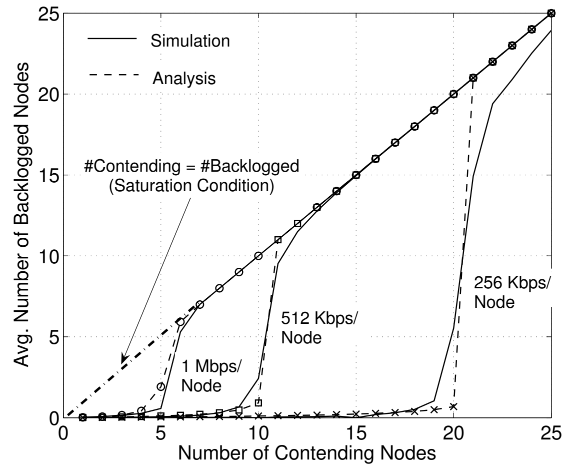


Figure 6: Average Number of Backlogged Nodes for different Arrival Rates

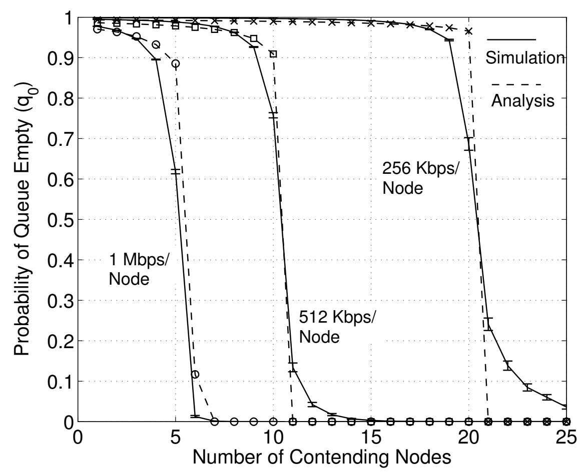


Figure 7: Comparison of Probability of Queue Empty for different arrival rates

The comparison of collision probability obtained from (2) and from simulations is shown in Figure 4. It can be seen from the figure that the collision probabilities rise rapidly after the number of nodes in the network increase beyond a certain threshold. On closer inspection, it can be seen that once the average aggregate arrivals to the network go beyond 5.5 Mbps, the collision probability increases suddenly. This is the saturation point of the network. It can be observed that the network reaches saturation at approximately 5 Nodes for 1 Mbps per node traffic. This can be observed for the other arrivals as well, 11 Nodes and 21 Nodes for 512 kbps per node and 256 kbps of traffic per node respectively. Hence, for 1500 byte packets, the saturation condition of the network is dominated by the aggregate traffic to the network and not by the number of contending nodes in the network. There is a close match between the simulation values and the values obtained from analysis. The expressions for γ, β and q_0 are not valid once the network reaches saturation condition as the queue becomes unstable (i.e., total arrivals are more than the total departures). Hence, in the saturation condition, the analysis degenerates to a simple saturation case analysis without queues as given in [10].

The collision probability observed for smaller packet sizes is shown in Figure 5. We fix the arrival rate of traffic to 256 kbps per node and vary the size of packets. It can be seen that even though the arrival rate is the same, smaller packet sizes lead to saturated network condition sooner. This happens as a result of heavy contention in the channel at smaller packet sizes. When smaller packet sizes are used keeping the data rate constant, the rate at which individual packet arrive at the queue increases. This leads to more frequent attempts to transmit in the network. This increased contention in the network leads to early saturation condition. It should be noted, that in the case with different packet sizes, the overheads due to collisions dominate the saturation effect.

Figure 6 shows the average number of backlogged nodes in the network at any given point of time. This number is calculated as $n(1-q_0)$, where q_0 is from (5). The network is in saturation condition if the average number of backlogged nodes are equal to the number of contending nodes. It can be seen that for higher loads the network reaches saturation sooner as compared to lightly loaded flows. This verifies the observation that the network is capacity limited by the aggregate arrivals and not by the number of nodes. We can use this result as noted in [11] to determine the TCP throughput by replacing the number of nodes in the network by the average number of backlogged nodes and performing a saturation analysis.

Figure 7 shows the probability of the queue being empty as the number of nodes in the network increase for different arrival rates. It can be observed, that the queues remain empty with probability more than 0.9 till the network reaches the saturation point. Once the saturation point is reached, the probability of queue being empty rapidly drops and stabilizes at 0. The saturation point is the same as observed in the case of collision probability γ. The analytical model developed by us is able to track the behavior of the queues accurately in the non-saturated region.

The comparison between the analysis and simulation results for Basic Access Method follow the same trend. The

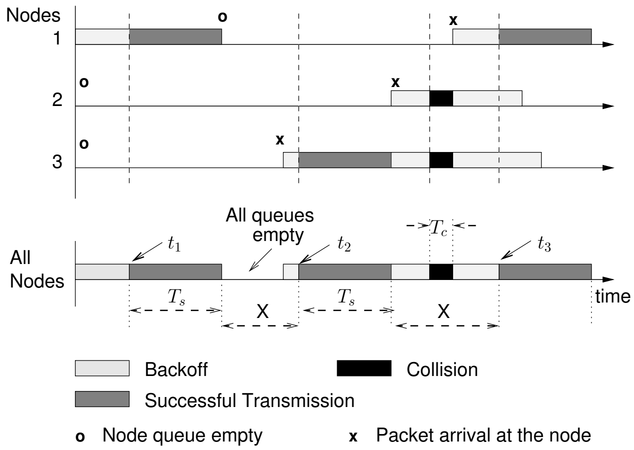


Figure 8: Time between two successful transmission attempts.

Basic Access Method does not involve sending of RTS-CTS packets before the transmission of DATA packets. Hence, apart from the times T_s and T_c for successful transmissions and collisions respectively, the rest of the analysis remains the same.

4. EXIT PROCESS OF AN IEEE 802.11 CELL

The exit process of an IEEE 802.11 cell is the time observed by an external entity between two successful packet transmissions in the cell irrespective of the source node. This definition is different from the service time of a node.

As illustrated in Figure 8, we are interested in the time intervals $(t_2 - t_1)$ and $(t_3 - t_2)$. It can be noted that the the minimum time between two successes is T_s, i.e., the time associated with the transmission of a successful packet. Also note that between two successful transmissions, there could be zero or more collisions. The other overheads include the time consumed in backoff count-down, collisions occurring in the network, and the time for which all the queues in the network are empty. Hence, the exit time is $T_s + X$, where X represents the time spent because of backoff, collisions and the idle time when all queues are empty. In Section 4.1 we derive expressions for $P(X = x)$ and compare them with QualNet simulation in Section 4.2.

4.1 Derivation of $P(X = x)$

Let A be the event that no node is backlogged. Then $P(A) = q_0^n$ and we write

$$P(X = x) = q_0^n P(X = x|A) + (1 - q_0^n)P(X = x|A^c). \tag{6}$$

When there is no backlog, the next transmission occurs as soon as a packet arrives. Thus

$$P(X = x|A) = \psi(1 - \psi)^{x-1}, \quad \psi = 1 - (1 - \lambda)^n. \tag{7}$$

So in order to completely characterize $P(X = x)$, we only have to find $P(X = x|A^c)$. We note that

$$P(X = x|A^c) = \sum_{i=0}^{\infty} P(X = x, S_i|A^c) \tag{8}$$

where S_i is the event that there are i failures before the success. The probability of attempt by at least one node in the network in a time slot is

$$\begin{aligned}\phi &= P(\text{At least one node attempts})\\ &= 1 - P(\text{No node attempts})\\ &= 1 - \sum_{l=0}^{n} \binom{n}{l} q_0^{n-l}(1 - q_0)^l(1 - \beta)^l.\end{aligned} \tag{9}$$

Then

$$\begin{aligned}P(X = x, S_0|A^c) &= \phi(1 - \phi)^{x-1}(1 - \gamma)\\ P(X = x, S_1|A^c) &= (x - T_c - 1)\phi(1 - \phi)^{x-T_c-1}\gamma\\ &\quad \times (1 - \phi)^{T_c-1} \times \phi(1 - \gamma)\\ &= (x - T_c - 1)\phi^2(1 - \phi)^{x-2}\gamma(1 - \gamma).\end{aligned} \tag{10}$$

In (10), $P(X = x, S_0|A^c)$ represents the probability of a successful transmission on the first attempt, i.e., no node attempts for $x - 1$ slots and a single node attempts in the remaining slot. $P(X = x, S_1|A^c)$, represents a situation as depicted in Figure 8 by time interval (t_2, t_3). There are no attempts by any node in $x - T_c - 1$ slots, one attempt that results in a collision and the other that results in a success. The collision transmission could start in any slot among the $x - T_c - 1$ slots. Also, during the collision transmission, $Tc - 1$ slots, no node attempts.

Since γ is usually small, we ignore the higher order terms in (8) and approximate $P(X = x|A^c)$ just with the above two terms. Thus from (6), (7), (8), and (10), we get a simple approximation to $P(X = x)$.

Now, the inter-exit time distribution is given by $T_s + X$, T_s is the constant time incurred in successful transmission of a packet derived in Section 3.3, and X has probability law derived above. Our analysis is valid for both basic access mechanism as well as the DCF mode of operation of IEEE 802.11 MAC protocol.

4.2 Comparison with Simulations

The time stamps for all packet transmission attempts, successes and collisions, are collected during the simulations to obtain exit times. The time between two successful transmissions is recorded as exit time. Since, all the analytical derivations are based on a slot time scale, we convert the exit time to slots. At the end of the simulation, the time between consecutive successes is computed and divided by the slot time (20 μsec) to obtain the exit times in terms of time slots. The parameters used in the simulations are given in Table 3. The simulations results presented in Figures 9-11 are for a single run of the experiment.

Figures 9 and 10 show the comparison between analysis and simulation for cumulative density function (CDF) of the exit times. The simulations have been performed for DCF mode of operation. It can be observed that as the number of nodes in the network increases, the network spends more time in the backlogged phase (with probability $1-q_0^n$), and lesser time in the idle phase waiting for packet arrivals (with probability q_0^n). Since the constant overhead for a successful packet transmission is 101 slots in DCF, as derived in Section 3.3, the observed CDF has a constant minimum overhead of roughly 100 slots between two successes. It can also be noted that the network reaches near saturation condition for fewer number of nodes in the case of 512 kbps per

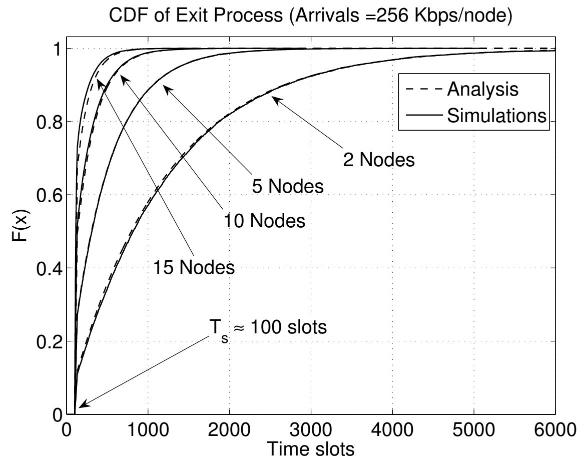


Figure 9: Comparison of Exit Time Distribution for varying number of contending nodes. Each node has an arrival rate of 256 kbps.

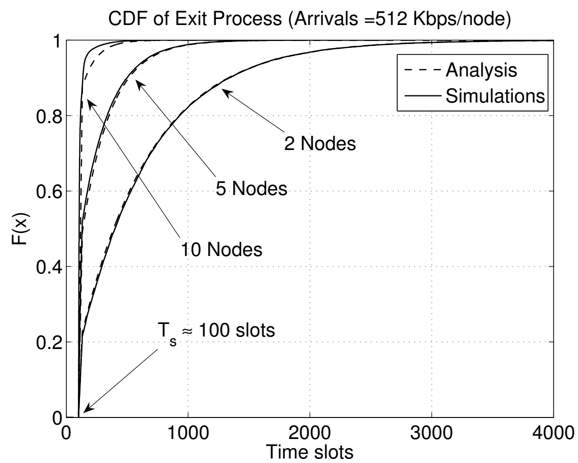


Figure 10: Comparison of Exit Time Distribution for varying number of contending nodes. Each node has an arrival rate of 512 kbps.

node arrival rate, i.e., 10 nodes instead of 15 nodes in case of 256 kbps per node.

Figure 11 shows the CDF for arrival rate of 1 Mbps per node. It can be seen that the network spends most of the time in backlogged phase and channel contention and less idle time even for very few nodes. This observation is in agreement with the saturation conditions observed in Figure 6. It can also be observed that at higher arrival rates, the maximum time spent in the idle state waiting for packet arrivals for 2 nodes is reduced from 6000 slots for 256 kbps/node traffic to 2000 slots for 1 Mbps/node traffic. The time spent in idle state for 2 nodes in the case of 512 kbps/node traffic is 4000 slots.

5. CONCLUSIONS AND FUTURE WORK

In this paper, we analyzed the performance of a non-saturated IEEE 802.11 based network. We incorporated the queuing behavior in the fixed point analysis and de-

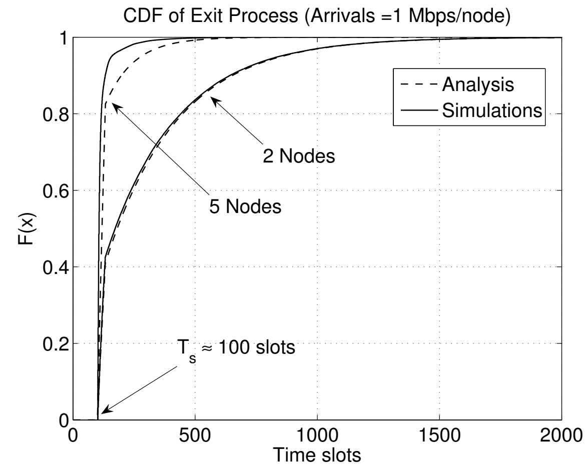


Figure 11: Comparison of Exit Time Distribution for varying number of contending nodes. Each node has an arrival rate of 1 Mbps.

rived inter-dependent relations for the queue being empty, collision probability, and packet transmission attempt. A key step in this derivation is the "mean-field" approximation used to correct for the scaling between the Backoff Time and the System Time. We also derived closed form approximations for the inter-exit time. Our analytical approximations agree closely with detailed QualNet simulations and form the basis for fast simulation of WLAN exit traffic in hybrid networks. For hybrid networks simulations, the traffic of the IEEE 802.11 WLANs at the access network level, that feeds traffic to the backbone network, can be replaced by random traffic generated by our model. The performance of such hybrid networks is currently under investigation.

6. ACKNOWLEDGMENTS

Punit Rathod's research is supported by the Infosys Fellowship Award from Infosys Technologies Limited, India. Abhay Karandikar's research is supported by the Tata Tele Services-IIT Bombay Center for Excellence in Telecommunication.

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